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Simple Lie Algebras in Characteristic 2 Recovered from Superalgebras and on the Notion of a Simple Finite Group

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Introduction

In what follows, unless mentioned to the contrary, k is an algebraically closed field, $\operatorname{char} k = p$; preliminaries, mostly preprinted so far, are given in Appendix.

The purpose of this paper is to formulate a conjecture describing simple finite-dimensional Lie algebras over k when $\operatorname{char} k = 2$; back it up with interesting examples (manifestly, until totally new ideas are incorporated, the conjecture is unprovable) and indicate connections with other problems. We find it interesting that the main tool is the theory of Lie superalgebras.

0.1. Around 1966 Kostrikin and Shafarevich found the following way to describe simple finite-dimensional Lie algebras over k. Their description suits restricted algebras best.

Recall that in char p the Leibnitz rule implies that if D is a derivation of an algebra then so is D^p. A Lie algebra in char p is called *restricted* or endowed with a *p-structure* if $(\operatorname{ad} x)^p$ is an inner derivation. For a centerless algebra the equation $(\operatorname{ad} x)^p = \operatorname{ad} y$ has a unique solution and this solution is denoted by $x^{[p]}$.

Let $\mathfrak{g}$ be a simple Lie algebra over $\mathbb{C}$ either finite-dimensional or vectory, i.e., of vector fields (known lately under unsuggestive and confusing name "Lie algebras of Cartan type"). In $\mathfrak{g}$, select a basis with respect to which the structure constants are integers, i.e., take a $\mathbb{Z}$-form $\mathfrak{g}_{\mathbb{Z}}$ of $\mathfrak{g}$.

RESTRICTED CONJECTURE (Kostrikin-Shafarevich). All simple finite-dimensional restricted Lie algebras over k are obtained for $p \geq 7$ as follows: tensor $\mathfrak{g}_{\mathbb{Z}}$ by k and select a simple subquotient of $\mathfrak{g}_{\mathbb{Z}} \otimes_{\mathbb{Z}} k$ (hereafter denoted by $\mathfrak{si}(\mathfrak{g})$).

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For $p > 7$ this conjecture is proved by Block and Willson, **[BW]**. We believe this is a matter of time to prove this conjecture for $p = 7$.

0.2. For nonrestricted algebras there were, contrarywise, so many examples which are not obtained in the above way that Block pessimistically wrote: "their completely explicit classification may be beyond reach" **[B]**. The known examples fall either into series of "twisted" versions of Lie algebras of derivations of the algebra of divided powers (**[KSh, BW, SF]**); or, for $p = 5$, constitute some bizzare examples found by Melikyan; or, for $p = 3$, are *deforms* (i.e., results of deformation) found by A. Rudakov and A. Kostrikin. For $p = 2$ there is classification of algebras possessing a Cartan matrix due to Kac and Weisfeiler **[WK]** and some nonrestricted examples found by Kaplansky **[K]**.

A way to describe the bulk of these extra examples (at least for $p > 2$) was suggested by Kostrikin who eagerly investigated a deformation for $p = 3$. He practically formulated the following

CONJECTURE FOR $p > 2$. All simple finite-dimensional Lie algebras over k for $p > 2$ are those of the form $\mathfrak{si}(\mathfrak{g})$, obtained as described above, or their deforms plus, perhaps, Melikyan's examples.

(Though recently (1988) Elsting (Kazan) has announced that Mekikyan's examples are not standard grading preserving deforms of a contact algebra, they may be deforms with respect to a nonstandard grading; anyway, they are constructed in exactly the same way as $\mathfrak{coect}(0/3)_*$, one of Shchepochkina's examples of exceptional simple vectory Lie superalgebras **[Shch, LS]**. This is our first indication of possible usefulness of comparing theories for superalgebras and Lie algebras in char p.)

The idea behind this conjecture is that deformations can be described very explicitly.

Deformations over k provide us with new phenomena: they intermix the conventional types of algebras. Examples are: the deformation (known to physicists as *quantization*) of the Poisson algebra, i.e., the Lie algebra of functions with Poisson bracket, into the Lie algebra of differential operators can be restricted over k to a deformation of $\mathfrak{sh}$ into $\mathfrak{psl}$; general vectory algebra can be deformed into a contact or hamiltonian one **[Kr]**, etc.

Notice however, that the abundance of deformations over k is deceiving. As shown by Kostrikin and Dzhumadildaev (see **[Kr]** and references therein),

the lesser p the more deformations a simple algebra has;

local deformations can usually be extended to global ones;

the deforms, corresponding to nontrivial deformations, usually (not just often!) are isomorphic to the initial algebra (to the frustration of the classifier).

0.3. Around 1973 Leites conjectured that to embrace all algebras for $p = 2$, not only those with Cartan matrix, one should incorporate superalgebras and started the study of the notion of restrictedness for Lie superalgebras

[L1], see also [L2, L3].

First, notice that *for $p = 2$ there are no Lie superalgebras, there are only $\mathbb{Z}/2$-graded Lie algebras.* Indeed, the modification of definitions required by the passage from zero characteristic to a prime one in case $p = 2$ only affects even elements. The only property which differentiates Lie superalgebras from $\mathbb{Z}/2$-graded Lie algebras is that the former can have a nonzero bracket of an odd element with itself. But since in the universal enveloping $[x, x] = 2x^2$ for any odd x, this difference between Lie algebras and Lie superalgebras disappears in char 2.

0.3.1. Digression. Note that since for $p = 3$ the modification of the definition of Lie superalgebra affects odd elements, there is a possibility of getting perfectly new examples of simple superalgebras only existing for $p = 3$. Namely, call a superalgebra over a field k of characteristic 3 a *3-Lie superalgebra* if the product in it is superskewcommutative and satisfies the Jacobi identity with respect to Sign Rule.

Clearly, the notion of 3-Lie superalgebra is, when $\operatorname{char} k = 3$, wider then that of a Lie superalgebra. Indeed a Lie superalgebra satisfies the Jacobi identity (minding Sign Rule), *and*,

$$(*) \qquad [x, [x, x]] = 0 \text{ for an odd element } x.$$

The identity $(*)$ is not a consequence of the Jacobi identity when $\operatorname{char} k = 3$.

PROBLEM. Are there examples of simple 3-Lie superalgebras which are not Lie superalgebras?

0.4. Restrictedness of superalgebras. The notion of restrictedness acquires new features when applied to Lie superalgebras for $\operatorname{char} k = p > 2$ and Lie algebras in $\operatorname{char} 2$.

For a superalgebra, in addition to the p-structure on $\mathfrak{g}_0$ we might have a $(p, -)$ and a *$(p, 2p)$-structure* on $\mathfrak{g}$, i.e. a p-structure on $\mathfrak{g}_0$ and either no structure or a $2p$-structure on $\mathfrak{g}_1$, respectively. If $p > 2$ the existence of a $(p, 2p)$-structure is equivalent to that of a p-structure on $\mathfrak{g}_0$, since the map $[2p]: \mathfrak{g}_1 \to \mathfrak{g}_0$ factors as $((\operatorname{ad} x(x))/2)^{[p]}$. Therefore for $p > 2$ either there is no p-structure or there is a $(p, 2p)$-structure.

A Lie algebra $\mathfrak{g}$ in $\operatorname{char} 2$, even a $\mathbb{Z}/2$-graded one, may possess a 2-structure not only on its even part. We could not rule out the possibility of $(2, -)$-structure.

PROBLEM. Does a $(2, -)$-structure exist?

0.5. Volichenko algebras. Recently Serganova listed simple nonhomogeneous subalgebras of all finite-dimensional and some vectory Lie superalgebras. Such nonhomogeneous subalgebras of Lie superalgebras are called *Volichenko* subalgebras [S]. Their list is discrete (meaning that it is reasonable). Speaking loosely, Volichenko algebras are unconventional deformations of Lie algebras. They exist and contribute to our list of $\operatorname{char} 2$ analogue of K-Sh conjecture as follows:

THEOREM. $\mathfrak{si}(\mathfrak{h})$, *where* $\mathfrak{h}$ *is a simple Volichenko subalgebras from* [S], *is a simple Lie algebra in char* 2.

0.6. CHAR 2 CONJECTURE. To obtain all (or at least restricted in some sense) simple finite-dimensional Lie algebras over k for $p = 2$ take for the input in Conjecture 0.2 above simple $\mathbb{Z}$-graded Lie algebras (or Lie superalgebras) of finite growth or Volichenko subalgebras of the latter.

To back up this conjecture we will produce several new series and exceptional examples; all the known examples fall into the conjecture.

0.7. Our interest in Lie algebras in char 2 was stirred by general idiosincracy for "super" and great expectations of utilitarian applications of our results in the singularity theory of real manifolds, where root systems over $\mathbb{Z}/2$ appear *deus ex machina* ([I]); after all, in char 2 a Lie algebra over $\mathbb{Z}/2$ is just a form of a Lie algebra over k.

0.8. The above char 2 conjecture is closely related to another conjecture Leites had made in 1972 shortly after having defined what is nowadays called superscheme [L1]. Slightly modified it reads as follows:

> *Over* GF(q), *Chevalley supergroup schemes corresponding to simple Lie superalgebras may produce new simple finite supergroups.*

How close are these supergroups to groups? Since for $p > 2$ the odd part of a Lie superalgebra can only generate (a part of) the radical of the group of Λ-points of its supergroup's superscheme, where Λ is a Grassmann superalgebra, there is no hope to obtain in this way a simple group. However, for $p = 2$ consideration of just GF(q)-points will get us some simple groups, though, hardly new ones.

We believe that since the supergroups corresponding to simple Lie superalgebras are *simple in the category of superschemes*, the groups of Λ-points of Chevalley supergroups corresponding to simple Lie superalgebras, though not simple themselves, are of great interest.

1. Integer bases in Lie superalgebras

For a Lie superalgebra $\mathfrak{g}$ *with a Cartan matrix* (a_{ij}) select a Chevalley basis, i.e. elements $X_i^{\pm}$ of degree ± 1 and $H_i = [X_i^+, X_j^-]$ (of degree 0) that generate $\mathfrak{g}$ and satisfy the relations $[X_i^+, X_j^-] = \delta_{ij}H_i$, $[H_i, X_j^{\pm}] = \pm a_{ij}X_j^{\pm}$, $(\operatorname{ad} X_i^{\pm})^{1-a_{ij}}(X_j^{\pm}) = 0$. There are several ways to normalize a Cartan matrix which may tell on after reduction mod p. A natural way described in [LS] is not the only natural one. If a simple root is not isotropic we can put on the diagonal of the Cartan matrix either 2 or, sometimes, 1; then the Cartan matrix of $\mathfrak{o}(2n+1)$ is indistinguishible from that of $\mathfrak{osp}(1/2n)$:

$$\begin{pmatrix} 2 & -1 & & & & \\ & & & & 0 & \\ -1 & 2 & & & & \\ & \ddots & \ddots & & \ddots & \\ & & -1 & 2 & & -1 \\ & 0 & & & & \\ & & & -2 & & 2 \end{pmatrix} \quad \begin{pmatrix} 2 & -1 & & & & \\ & & & & 0 & \\ -1 & 2 & & & & \\ & \ddots & \ddots & & \ddots & \\ & & -1 & 2 & & -1 \\ & 0 & & & & \\ & & & -1 & & 1 \end{pmatrix}$$

For vectory Lie superalgebras integer bases are associated with $\mathbb{Z}$-forms in $\mathbb{C}[U]$, $U = (U_1, \dots, U_a)$. For a multiindex r set

$$u^r = \prod u_i^r i = \prod (U_i^r i)/r_i!$$

For a set of integers $m = (m_1, \dots, m_a)$ with $m_1 \geq \cdots \geq m_a \geq 0$ denote by $k[u; m]$ the k-span of $\{u^r : r_i < p_i^m\}$. Since $u^r u^s = C_s^{r+s} u^{r+s}$, then $k[u; m]$ is a subalgebra of $k[u]$. The algebra $k[u]$ and its subalgebras $k[u; m]$ are called the *algebras of divided powers.* Thus, $\mathbb{Z}$-forms of vectory superalgebras acquire one more parameter, which concerns only even indeterminates. Denote by

$$\mathfrak{vect}(a; m/b; r) = \mathfrak{der}\, k[u; m], \quad \text{where } \deg u_i = r_i, \ \deg \xi_j = r_{a+j},$$

the *general vectory Lie algebra* (for the standard grading we skip r).

2. Examples

2.1. Algebras with Cartan matrix. Both $\mathfrak{sl}(2)$ and $\mathfrak{sl}(1/1)$ turn over k into one Lie algebra (denote it $\mathfrak{sl}(2)$) with Cartan matrix (0); while $\mathfrak{si}(\mathfrak{osp}(1/2)) = \mathfrak{o}(3)_{\mathbb{Z}} \otimes k$, let (1), the matrix with one entry—1, be its Cartan matrix.

In general for a Lie superalgebra over $\mathbb{C}$ with Cartan matrix take Chevalley generators X_i, $1 \leq i \leq n$, with respect to a basis (system of simple roots) and, unlike [S], normalize the Cartan matrix so that on the diagonal stands 0 or $\min(1, 2)$. Then construct a simple Lie algebra over k as above.

The results are summarized in Table 2 which also contains $\mathfrak{si}$ of some Lie superalgebras without Cartan matrix (note that $\mathfrak{si}(\mathfrak{psq}(n)) = \mathfrak{si}(\mathfrak{sl}(n))$; $\mathfrak{si}(\mathfrak{spe}(n)) = \mathfrak{si}(\mathfrak{o}(2n)))$.

Remark. The family $\mathfrak{d}(\alpha)$ gives rise to a deformation of $\mathfrak{psl}(4)$. The dimension of the deform is miscalculated in [WK]: consider the exact sequence

$$0 \to \mathfrak{psl}(2/2) \to \mathfrak{d}(-1) \to \mathfrak{sl}(2) \to 0$$

and notice that over k this sequence may be deformed, which gives rise to a nontrivial deformation of the subalgebra $\mathfrak{psl}(4)$: upper triangular matrices; then $E^2(\mathrm{id})$ is an irreducible submodule in $S^2(\mathrm{id})$ and the quotient is the trivial n-dimensional one. See Figure 1.

In the remainder of the table $\mathfrak{g}_i = \mathfrak{g}_i^*$ as $\mathfrak{g}_0$-modules.

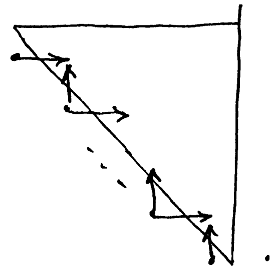

FIGURE 1

TABLE 1. $\mathbb{Z}$-gradings of some Lie superalgebras over $\mathbb{C}$ and their $\mathfrak{si}$.

$\mathfrak{A}$	$\mathfrak{sp}(2n)$	$\mathfrak{pe}^{sy}(n)$	$\mathfrak{pe}^{sk}(n)$	$\mathfrak{spe}^{sy}(n)$	$\mathfrak{o}(2n)$	$\mathfrak{si}(\mathfrak{g})$
$\mathfrak{A}_{-1}$	$S^2(\mathrm{id})^*$	$\Lambda^2(\mathrm{id})^*$	$S^2(\mathrm{id})^*$	$\Lambda^2(\mathrm{id})^*$	$\Lambda^2(\mathrm{id})^*$	$\Lambda^2(\mathrm{id})^*$
$\mathfrak{A}_0$	$\mathfrak{gl}(n)$	$\mathfrak{gl}(n)$	$\mathfrak{gl}(n)$	$\mathfrak{sl}(n)$	$\mathfrak{gl}(n)$	$\mathfrak{sl}(n)$ or $\mathfrak{psl}(n)$
$\mathfrak{A}_1$	$S^2(\mathrm{id})$	$S^2(\mathrm{id})$	$\Lambda^2(\mathrm{id})$	$S^2(\mathrm{id})$	$\Lambda^2(\mathrm{id})$	$\Lambda^2(\mathrm{id})$

$\mathfrak{A}$	$\mathfrak{osp}(2m/2n)$	$\mathfrak{si}(\mathfrak{A})$
$\mathfrak{A}_{-1}$	$E^2(\mathrm{id})^*$	$\Lambda^2(\mathrm{id}^*)$
$\mathfrak{A}_0$	$\mathfrak{gl}(m/n)$	$(\mathfrak{p})\mathfrak{sl}(m+n)$
$\mathfrak{A}_1$	$E^2(\mathrm{id})$	$\Lambda^2(\mathrm{id})$

$\mathfrak{A}$	$\mathfrak{o}(2n+1)$, $n>1$	$\mathfrak{osp}(1/2n)$, $n>1$	$\mathfrak{si}(\mathfrak{A})$	$\mathfrak{o}(3)$, $\mathfrak{osp}(1/2)$	$\mathfrak{si}(\mathfrak{A})$
$\mathfrak{A}_{-2}$	$\Lambda^2(\mathrm{id})^*$	$S^2(\mathrm{id})^*$	$\Lambda^2(\mathrm{id})$	id^*	–
$\mathfrak{A}_{-1}$	id^*	$\pi(\mathrm{id}^*)$	id	id^*	id^*
$\mathfrak{A}_0$	$\mathfrak{gl}(n)$	$\mathfrak{gl}(n)$	$\mathfrak{sl}(n)$	$\mathfrak{gl}(1)$	$\mathfrak{gl}(1)$

The exterior algebra of a superspace is denoted by E, that of the space by Λ; thus E of a purely even space is S.

2.2. Vectory Lie algebras. In this note we confine ourselves to truncated polynomials leaving the case of divided powers till a detailed version of this text, cf. [KL]. Let $\mathfrak{g}'=[\mathfrak{g},\mathfrak{g}]$, $\mathfrak{g}^{(i)}=[\mathfrak{g}^{(i-1)},\mathfrak{g}^{(i-1)}]$.

First note that, clearly,

$$\mathfrak{si}(\mathfrak{vect}(m/n))=\mathfrak{si}(\mathfrak{vect}(m+n))=\mathfrak{der}\,k[\xi_1,\dots,\xi_{m+n}],$$

$$\mathfrak{si}(\mathfrak{svect}(m/n))=\mathfrak{si}(\mathfrak{svect}(m+n))=\{D\in\mathfrak{vect}(m+n)\colon \operatorname{div}D=0\}'$$

2.2.1. LEMMA. *Let* $2m_1+n_1=2m_2+n_2>3$ *and* $n_1n_2\neq 0$. *Then*

1) $\mathfrak{si}(\mathfrak{h}(2m_1/n_1))=\mathfrak{si}(\mathfrak{h}(2m_2/n_2))$;

2) *there exists a deformation of* $\mathfrak{si}(\mathfrak{h}(2m/0))$ *into* $\mathfrak{si}(\mathfrak{h}(2m-2/2))$ *and the nonzero values of the corresponding cocycle are*:

$$c(h_{Ap_n},h_{Bp_n})=c(h_{Aq_n},h_{Bq_n})=h_{AB},$$

$$c(h_{Ap_n},h_{Bp_nq_n})=h_{ABq_n},\qquad c(h_{Aq_n},h_{Bp_nq_n})=h_{ABp_n},$$

where A *and* B *are monomials independent of* p_n *and* q_n.

2.2.2. LEMMA. 1) $\mathfrak{si}(\mathfrak{t}(2n+1/m)) = \mathfrak{si}(\mathfrak{t}(2n-1/m+2))$ *for* $m > 0$;

2) *there exists a deformation of* $\mathfrak{si}(\mathfrak{h}(2n/2m+1)$ *into* $\mathfrak{si}(\mathfrak{t}(2n+1/2m))$ *and the nonzero values of the corresponding cocycle* $c = c_1 + c_2$ *are*

$$\begin{aligned}
&c_1(K_A, K_{B\xi_{2m+1}}) = (\deg K_A)K_{AB},\\
&c_1(K_{A\xi_{2m+1}}, K_{B\xi_{2m+1}}) = (\deg K_A + \deg K_A)K_{AB\xi_{2m+1}},\\
&c_2(K_{A\xi_{2m+1}}, K_{B\xi_{2m+1}}) = K_{AB}.
\end{aligned}$$

3) $\mathfrak{si}(\mathfrak{t}(2n+1/0)) = \mathfrak{si}(\mathfrak{h}(2n/0)) = \mathfrak{si}(\mathfrak{le}(2n/0)) = \mathfrak{si}(\mathfrak{m}(n))$

4) $\mathfrak{si}(\mathfrak{sle}(n)) = \mathfrak{si}(\mathfrak{b}_1(n))$, $\dim \mathfrak{si}(\mathfrak{sle}(n)) = 2^{2n-1} - 2^{n-1} - 2$.

2.2.3. LEMMA. 1) $\mathfrak{si}(\mathfrak{coect}(0/3)_*^m) = \mathfrak{si}(\mathfrak{h}(6/0))$;

2) $\dim \mathfrak{si}(\mathfrak{coect}(0/3)_*^m) = 59$.

REMARK. It seems that $\mathfrak{si}(\mathfrak{sle}(n))$, their deforms, and the deforms of their deforms give new series of simple Lie algebras; $\mathfrak{si}(\mathfrak{coect}(0/3)_*^m)$ is an example of a new exceptional simple Lie algebra.

3. Existence of 2-structure, $(2, -)$-and $(2, 4)$-structures

THEOREM. *If* $\mathfrak{g}$ *is a Lie algebra over* k *with Dynkin-Kac diagram without* $\bullet$ *then* $\mathfrak{g}$ *has a* 2*-structure, otherwise it has a* $(2, 4)$*-structure.*

The vectory Lie algebras in the above examples posses a 2*-structure, except for* $\mathfrak{si}(\mathfrak{t}(2n+1/m))$ *for* $m \neq 0$; $\mathfrak{si}(\mathfrak{h}(2n/m))$ *for* $nm \neq 0$ *and* $\mathfrak{si}(\mathfrak{h}(0/m))$ *which possess a* $(2, 4)$*-structure.*

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Some of the results of this paper had been announced in [KL], some were preprinted (very informally) at Harvard in 1987.

TABLE 2. Bases (systems of simple roots) for $p = 2$

$\mathfrak{A}$	its Cartan matrix	its Dynkin diagram	diagram of $\mathfrak{si}(\mathfrak{A})$
$\mathfrak{sl}(2)$	(2)	○	
$\mathfrak{sl}(1/2)$	(0)	⊗	⊗
$\mathfrak{osp}(1/2)$	(1)	•	•
$\mathfrak{o}(3)$	(1)	○	

$\mathfrak{A}$	its matrix or diagram			
$\mathfrak{d}(\alpha)$	$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & 1+\alpha & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & -\alpha & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & -\alpha & 0 \\ 1 & 0 & -1 \\ 0 & 1+\alpha & 2 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 & -1-\alpha \\ 1 & 0 & \alpha \\ -1-\alpha & \alpha & 0 \end{pmatrix}$
$\mathfrak{si}(\mathfrak{d}(\alpha))$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1+\alpha & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & \alpha & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & \alpha & 0 \\ 1 & 0 & 1 \\ 0 & 1+\alpha & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 1+\alpha \\ 1 & 0 & \alpha \\ 1+\alpha & \alpha & 0 \end{pmatrix}$
$\mathfrak{A}_2$	$\begin{pmatrix} 0 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 2 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -3 & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & -1 & -2 \\ -1 & 0 & 3 \\ -2 & 3 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & -1 \\ 0 & -3 & 2 \end{pmatrix}$

$\mathfrak{A}$	its diagram	
$\mathfrak{e}_n$	0 − ○ − ○ − ⋯ − ○ | ○	
$\mathfrak{si}(e_n)$	⊗ − ⊗ − ⊗ − ⋯ − ⊗ | ○ ⊗	
$\mathfrak{A}_2$	0 ⇛ ○	
$\mathfrak{si}(\mathfrak{A}_2)$	⊗ − ⊗	
$\mathfrak{f}_4$	○ − ○ ⇒ ○ − ○	
$\mathfrak{si}(\mathfrak{f}_4)$	⊗ − ⊗ ⊗ − ⊗	
$\mathfrak{si}(\mathfrak{A}_2)$	⊗ − ⊗ − ⊗	⊗ − ⊗ •
$\mathfrak{Ab}_3$	○ ⇛ ⊗ − ○ ⇐ ⊗ − ⊗ ⇐ ○ (⊗ below, ≡ and // bonds) ○ ⇒ ⊗ = ⊗ (○ below)	○ − ○ ⇒ ○ − ⊗ ○ − ○ ⇒ ⊗ − ⊗
$\mathfrak{si}(\mathfrak{ab}_3)$	⊗ − ⊗ − ⊗ − •	⊗ − ⊗ ⊗ − ⊗

$\mathfrak{A}$	its diagram	diagram of $\mathfrak{si}(A)$
$\mathfrak{osp}(1/2n)$ $\mathfrak{osp}(2m+1/2n)$	○ − ⋯ − ○ − • • − ⋯ − • − • • − ⋯ − • − ○	⊗ − ⋯ − ⊗ − •
$\mathfrak{o}(2n+1)$	○ − ⋯ − ○ ⇐ ○	
$\mathfrak{osp}(2n/2n)$	• − ⋯ • − • ⇐ ○	
	• − ⋯ − • < ○, ○ • − ⋯ − • < ⊗ ‖ ⊗	⊗ − ⋯ − ⊗ < ⊗, ⊗
	• − ⋯ − • ⇒ ○	⊗ − ⋯ − ⊗ ⊗

Bibliography

[ALSh] D. Alekseevsky, D. Leites and I. Shchepochkina, *Examples of simple Lie superalgebras of vector fields*, C. R. Acad. Bulgare Sci. **33** (1980), 1187–1190.

[B] R. E. Block, *The classification problem for simple Lie algebras in characteristic* p, Lecture Notes in Math., vol. 933, Springer-Verlag, Berlin and New York, 1982, pp. 38–56.

[BW] R. E. Block and R. L. Wilson, *Classification of the restricted simple Lie algebras*, J. Algebra, **114** (1988).

[I] G. Il′uta, *On monodromy groups and Dynkin diagrams*, [L4, n13].

[K] V. Kac, *Lie superalgebras*, Adv. Math. **26** (1977), 8–96.

[Kp] I. Kaplansky, *Some simple Lie algebras in characteristic* 2, Lecture Notes in Math., vol. 933, Springer-Verlag, Berlin and New York, 1982, pp. 127–129.

[KL] Yu. Kochetkov and D. Leites, *Lie superalgebras and simple Lie algebras in char* 2, VINITI depositions, 1986–dep

[Ko] Yu. Kotchetkoff, *Deformations des superalgebres de Buttin et quantification*, C. R. Acad. Sci. Paris, t-299 serie 1, N 14, 1984, 643–645.

[Kr] A. I. Kostrikin, *Lie algebras and finite groups*, Trans. Steklov Math. Inst., vol. 168, 1984, 132–156. (Russian)

[L2] D. Leites, *New Lie superalgebras and mechanics*, Soviet Math. Dokl.

[L3] ____, *Supermanifold theory*, Karelia branch of the USSR Acad. of Sci., Petrozavodsk, 1983; an expanded version in 7 volumes to be published by Kluwer is meanwhile preprinted as [L4].

[L4] ____, ed. Seminar on Supermanifolds, no. 1–35. Reports of Dept. of Math. of Stockholm Univ. 1986–89.

[L5] ____, *Lie superalgebras*, JOSMAR, 30, no. 6, 1984, -2513.

[LS] D. Leites and I. Shchepochkina, *Classical Lie superalgebras*, [L4, n. 31].

[P] A. Premet, *Algebraic groups associated with Lie p-algebras of Cartan type*, Math. USSR-Sb. **50** (1985), 85–97. (Russian)

[Shch] I. Shchepochkina, *Exceptional simple Lie superalgebras of vector fields.* C.R. Acad. Bulgare. Sci. **36** (1983), 313–314.

[S] V. Serganova, *Simple Volichenko algebras*, [L4, n34] and these Proceedings.

[SF] H. Strade and R. Farnsteiner, *Modular Lie algebras and their representations*, M. Dekker, New York, 1988.