Classical superspaces and related structures

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Introduction

The main object in the study of Riemannian geometry is (properties of) the Riemann tensor which, in turn, splits into the Weyl tensor, Ricci tensor and scalar curvature. The word "splits" above means that at every point of the Riemannian manifold M^n the space of values of the Riemann tensor constitutes an O(n)-module which is the sum of three irreducible components (unless n =4 when the Weyl tensor aditionally splits into 2 components).

More genearlly, let G be any group, not necessarily O(n). In what follows we recall definition of *G-structure* on a manifold and of (the space of) its *structure functions* (SFs) which are obstructions to integrability or, in other words, to possibility of flattening the G-structure. Riemannian tensor is an example of SF. Among the most known (or popular of recent) examples of G-structures are:

- an almost *conformal structure*, $G = O(n)\times\mathbb{R}^*$, SF are called the *Weyl tensors*;
- Penrose' *twistor* theory, $G = SU(2)\times SU(2)\times\mathbb{C}^*$, SF -- the *Penrose tensor* -- splits into 2 components whose sections are called "*α-forms*" and "*β-forms*";
- an almost *complex structure*, $G = GL(n;\mathbb{C}) \subset GL(2n;\mathbb{R})$, SF is called the *Nijenhuis tensor*;
- an almost *symplectic structure*, G = Sp(2n), (no accepted name for SF).

The first two examples are examples of a "conformal" structure which preserves a tensor up to a scalar. In several versions of a very lucid paper [G] Goncharov calculated (among other things) all SF for all structures with a simple group of conformal transformations, whose subgroup of *linear* transformations is the reductive part of the stabilizer of a point of the space and is the "G" which determines the G-structure on the manifold. Remarkably, Goncharov's examples correspond precizely to the classical spaces, i.e. irreducible compact Hermitian symmetric spaces (CHSS). Goncharov did not, however, write down the highest weights of irreducible components of SFs; this is done in [LPS1] and some of these calculations are interpreted as leading to generalized Einstein equation.

In this talk we advertize results (mostly due to E.Poletaeva) of calculating SF (and interpretation of them) for classical superspaces who are defined and partly listed in [S] and [L2] (see also [V], containing interesting papers on supergravity and where curved supergrassmannians are introduced). The problem was raised in [L2], cf. [L4], and the above examples are now superized in [P] and [LPS]. The passage to supermanifolds naturally hints to widen the usual approach to SFs in order to embrace at least the following cases:

- 2 types of infinite dimensional generalizations of Riemannian geometry connected with: (1) string theories of physics (these infinite dimensional examples have no analogues on manifolds because they require no less then **three** odd coordinates of the superstring; the list of corresponding hermitian superspaces deduced from [S] is given in [L2]; dual pairs, etc. will be considered elsewhere) and (2) Kac-Moody (super) algebras (see Table 5);
- the G-structures of the N-extended Minkowski superspace: the tangent space to the Minkovski superspace for $N\neq 0$ is naturally endowed with a 2-step nilpotent Lie superalgebra structure that highly resembles the contact structure on a manifold. We start studying such structures in earnest in [LPS2], compair our approach with that of the GIKOS group lead by V.I. Ogievetsky. More generally, we shall calculate SF for the G-structures of the type corresponding to any "flag variety", not just Grassmannians, particular at that, see Table 1.

Elsewhere we will generalize the machinery of Jordan algebras, so useful in the study of geometry of CHSSs [Mc], to the cases we consider (this is Vinel's thesis).

Can programmers help? A good part of the calculations we need are very simple (to calculate cohomology is to solve systems of linear equations [F]). Still, though the number of papers on supergravity is counted by thousands (see reviews in our bibliography, of which [OS3], [WB], [We] are easy to understand) there is remarkably small progress in actual calculations (cf. mathematical papers [Sch], [RSh], [Me]). It is yet unclear what are all supergravities for N>1. The reason to that: the calculations *are* voluminous besides, these calculations also have to be "glued" in an answer and there are no rules for doing so, cf. [P4]. Thus the problem is a challenge for a computer scientist, our calculations, together with [LP1] and [P1-4], illustrate [LP2]. For our cohomology of our infinite dimensional Lie (super)algebras there are NO recipes at all (not even from Feigin-Fuchs nor Roger [FF]).

In this text we deal with linear algebra: at a point. The global geometry, practically not investigated, is nontrivial, cf. [M], [MV].

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Preliminaries

Terminological conventions. 1) A $\mathfrak{g}$-module V with highest weight ξ and *even* highest vector will be denoted by V_ξ or $R(\xi)$. An irreducible module with highest weight $\Sigma a_i\pi_i$, where π_i is the i-th fundamental weight, will be denoted sometimes by its numerical labels $R(\Sigma a_i; a)$ the highest weight with respect to the center of $\mathfrak{g}$ stands after semicolon, cf.[OV], Reference Chapter.

2) Let $\mathfrak{c}\mathfrak{g}$ denote the trivial central "extent" (the result of the extention) of a Lie (super)algebra $\mathfrak{g}$; let $\mathfrak{p}$ stand for projectivization (as in $\mathfrak{psl}$, $\mathfrak{pq}$) and $\mathfrak{s}$ for "trace"-less part (as in $\mathfrak{sl}$, $\mathfrak{sq}$, $\mathfrak{sh}$).

0.1. Structure functions. Let us retell some of Goncharov's results ([G]) and recall definitions ([St]).

Let M be a manifold of dimension n over a field $\mathbb{K}$; think $\mathbb{K} = \mathbb{C}$ (or $\mathbb{R}$). Let F(M) be the frame bundle over M, i.e. the canonical principal GL(n; $\mathbb{K}$)-bundle. Let $G \subset GL(n; \mathbb{K})$ be a Lie group. A *G-structure* on M is reduction of the frame bundle to the principal G-bundle corresponding to inclusion $G \subset GL(n; \mathbb{K})$, i.e. a *G-structure* is the possibility to select transition functions so that their values belong to G.

The simplest G-structure is the *flat* G-structure defined as follows. Let V be $\mathbb{K}^n$ with a fixed frame. Consider the bundle over V whose fiber over $v \in V$ consists of all frames obtained from the fixed one under the G-action, V being identified with T_vV.

Obstructions to identification of the k-th infinitesimal neighbourhood of a point $m \in M$ on a manifold M with G-structure and that of a point of the flat manifold V with the above G-structure are called *structure functions of order k*. Such an identification, is possible provided all structure functions of lesser orders vanish.

Proposition. ([St]). *SFs of order k are elements from the space of* (k,2)-*th Spencer cohomology.*

Recall definition of the Spencer cochain complex. Let S^i denote the operator of the i-th symmetric power. Set $\mathfrak{g}_{-1} = T_mM$, $\mathfrak{g}_0 = \mathfrak{g} = \mathrm{Lie}(G)$ and for i > 0 put:

$$(\mathfrak{g}_{-1}, \mathfrak{g}_0)_* = \oplus_{i \geq -1} \mathfrak{g}_i, \text{ where } \mathfrak{g}_i = \{X \in \mathrm{Hom}(\mathfrak{g}_{-1}, \mathfrak{g}_{i-1}): \ X(v)(w,...) = X(w)(v,...)$$

for any $v,w \in \mathfrak{g}_{-1}\} = S^i(\mathfrak{g}_{-1})^* \otimes \mathfrak{g}_0 \cap S^{i+1}(\mathfrak{g}_{-1})^* \otimes \mathfrak{g}_{-1}$.

Suppose that

the $\mathfrak{g}_0$*-module* $\mathfrak{g}_{-1}$ *is faithful.* (0.1)

Then, clearly, $(\mathfrak{g}_{-1}, \mathfrak{g}_0)_* \subset \mathfrak{vect}(n) = \mathfrak{der}\ \mathbb{K}[[x_1,..., x_n]]$, where $n = \dim \mathfrak{g}_{-1}$. It is subject to an easy verification that the Lie algebra structure on $\mathfrak{vect}(n)$ induces a Lie algebra structure on

$(\mathfrak{g}_{-1}, \mathfrak{g}_0)_*$. The Lie algebra $(\mathfrak{g}_{-1}, \mathfrak{g}_0)_*$, usually abbreviated to $\mathfrak{g}_*$, will be called *Cartan's prolong* (the result of *Cartan prolongation*) of the pair $(\mathfrak{g}_{-1}, \mathfrak{g}_0)$.

Let E^i be the operator of the i-th exterior power; set $C^{k,s}_{\mathfrak{g}_*} = \mathfrak{g}_{k-s} \otimes E^s(\mathfrak{g}_{-1}^*)$; usually we drop the subscript or at least indicate only $\mathfrak{g}_0$. Define the differential $\partial_s: C^{k,s} \to C^{k-1,s+1}$ setting for any $v_1, ..., v_{s+1} \in V$ (as always, the slot with the hatted variable is ignored):

$$(\partial_s f)(v_1, ..., v_{s+1}) = \Sigma(-1)^i f(v_1, ..., \hat{v}_{s+1-i}, ..., v_{s+1})(v_{s+1-i})$$

As usual, $\partial_s \partial_{s+1} = 0$, the homology of this complex is called *Spencer cohomology* of $(\mathfrak{g}_{-1}, \mathfrak{g}_0)_*$.

0.2. Case of simple $\mathfrak{g}_*$ over $\mathbb{C}$. The following remarkable fact, though known to experts, is seldom formulated explicitely:

Proposition. *Let $\mathbb{K} = \mathbb{C}$, $\mathfrak{g}_* = (\mathfrak{g}_{-1}, \mathfrak{g}_0)_*$ be simple. Then only the following cases are possible:*

1) $\mathfrak{g}_2 \neq 0$ and then $\mathfrak{g}_*$ is either $\mathfrak{vect}(n)$ or its special subalgebra $\mathfrak{svect}(n)$ of divergence-free vector fields, or its subalgebra $\mathfrak{h}(2n)$ of hamiltonian fields;

2) $\mathfrak{g}_2 = 0$, $\mathfrak{g}_1 \neq 0$ then $\mathfrak{g}_*$ is the Lie algebra of the complex Lie group of automorphisms of a CHSS (see above).

Proposition explains the reason of imposing the restriction (0.1) if we wish $\mathfrak{g}_*$ to be simple. Otherwise, or on supermanifolds, where the analogue of Proposition does not imply similar restriction, we have to (and do) broaden the notion of Cartan prolong to be able to get rid of restriction (0.1).

When $\mathfrak{g}_*$ is a simple finite-dimensional Lie algebra over $\mathbb{C}$ computation of structure functions becomes an easy corollary of the Borel-Weyl-Bott-... (BWB) theorem, cf. [G]. Indeed, by definition $\oplus_k H^{k,2}_{\mathfrak{g}_*} = H^2(\mathfrak{g}_{-1}; \mathfrak{g}_*)$ and by the BWB theorem $H^2(\mathfrak{g}_{-1}; \mathfrak{g}_*)$, as $\mathfrak{g}$-module, has as many components as $H^2(\mathfrak{g}_{-1})$ which, thanks to commutativity of $\mathfrak{g}_{-1}$, is just $E^2(\mathfrak{g}_{-1})$; the highest weights of these modules, as explained in [G], are also deducible from the theorem. However, [G] pityfully lacks this deduction, see [LP1] and [LPS1] where it is given with interesting interpretations.

Let us also immediately calculate SF corresponding to case 1) of Proposition: we did not find these calculations in the literature. Note that vanishing of SF for $\mathfrak{g}_* = \mathfrak{vect}$ and $\mathfrak{k}$ (see 0.5) follows from the projectivity of $\mathfrak{g}_*$ as $\mathfrak{g}_0$-modules and properties of cohomology of coinduced modules [F]. In what follows $R(\Sigma a_i \pi_i)$ denotes the irreducible $\mathfrak{g}_0$-module. The classical spaces are listed in Table 1 and some of them are bapthized for convenience of further references.

Theorem. 1)(Serre [St]). *In case 1) of Proposition structure functions can only be of order 1.*

a) $H^2(\mathfrak{g}_{-1}; \mathfrak{g}_*) = 0$ *for* $\mathfrak{g}_* = \mathfrak{vect}(n)$ *and* $\mathfrak{svect}(m)$, $m>2$;

b) $H^2(\mathfrak{g}_{-1}; \mathfrak{g}_*) = R(\pi_3) \oplus R(\pi_1)$ *for* $\mathfrak{g}_* = \mathfrak{h}(2n)$, $n>1$;

$H^2(\mathfrak{g}_{-1}; \mathfrak{g}_*) = R(\pi_1)$ *for* $\mathfrak{g}_* = \mathfrak{h}(2)$.

2)(Goncharov [G]). *SFs of* Q_3 *are of order 3 and constitute* $R(4\pi_1)$. *SF for Grassmannian* Gr_m^{m+n} (when neither m nor n is 1, i.e. Gr is not a projective space) *is the direct sum of two components whose weights and orders are as follows:*

Let $A = R(2, 0, ..., 0, -1) \otimes R(1, 0, ..., 0, -1, -1)$, $B = R(1, 1, 0, ..., 0, -1) \otimes R(1, 0, ..., 0, -2)$.

Then if $mn \neq 4$ both A and B are of order 1;
if $m = 2, n \neq 2$ A is of order 2 and B of order 1;
if $n = 2, m \neq 2$ A is of order 1 and B of order 2;
if $n = m = 2$ both A and B are of order 2.

SF of G-structures of the rest of the classical CHSSs are the following irreducible $\mathfrak{g}_0$-modules whose order is 1 (recall that $Q_4 = Gr_2^4$)*:*

CHSS	$\mathbb{P}^n$	OGr_m	LGr_m	Q_n , n>4
weight of SF	-	$E^2(E^2(V^*))\otimes V$	$E^2(S^2(V^*))\otimes V$	$E^2(V^*)\otimes V$

$E_6/SO(10)xU(1)$	$E_7/E_6xU(1)$
$E^2(R(\pi_5)^*))\otimes R(\pi_5)$	$E^2(R(\pi_1)^*))\otimes R(\pi_1)$

0.3. SF for reduced structures. In [G] Goncharov considered conformal structures. SF for the corresponding generalizations of the Riemannian structure, i.e. when $\mathfrak{g}_0$ is the semisimple part $^\wedge\mathfrak{g}$ of $\mathfrak{g}$ = Lie (G), seem to be more difficult to compute because in these cases $(\mathfrak{g}_{-1}, \mathfrak{g}_0)_* = \mathfrak{g}_- \oplus \mathfrak{g}_0$ and the BWB-theorem does not work. Fortunately, the following statement, a direct corollary of definitions, holds.

Proposition ([G], Th.4.7). For $\mathfrak{g}_0 = {}^\wedge\mathfrak{g}$ and $\mathfrak{g}$ SF of order 1 are the same and SF of order 2 for $\mathfrak{g}_0 = {}^\wedge\mathfrak{g}$ are $S^2(\mathfrak{g}_1) = S^2(\mathfrak{g}_{-1}{}^*)$. (There are clearly no SF of order 3 for $\mathfrak{g}_0 = {}^\wedge\mathfrak{g}$).

Example: Riemannian geometry. Let G = O(n). In this case $\mathfrak{g}_1 = \mathfrak{g}_{-1}$ and in $S^2(\mathfrak{g}_{-1})$ a 1-dimensional subspace is distinguished; the sections through this subspace constitute a Riemannian metric g on M. (The habitual way to determine a metric on M is via a symmetric matrix, but actually this is just one scalar matrix-valued function.) The values of the Riemannian tensor at a point of M constitute an O(n)-module $H^2(\mathfrak{g}_{-1}; \mathfrak{g}_*)$ which contains a trivial component whose arbitrary section will be denoted by R. What is important, this trivial component is realised by Proposition as a submodule in $S^2(\mathfrak{g}_{-1})$. Thus, we have two matrix-valued functions: g and R each being a section of the trivial $\mathfrak{g}_0$-module. What is more natural than to require their ratio to be a constant (rather than a function)?

$$R = \lambda g, \text{ where } \lambda \in \mathbb{R}. \qquad (EE_0)$$

Recall that the Levi-Civita connection is the unique symmetric affine connection compatible with the metric. Let now t be the structure function (sum of its components belonging to the distinct irreducible O(n)-modules that constitute $H^2(\mathfrak{g}_{-1}; \mathfrak{g}_*)$) corresponding to the Levi-Civita connection; the process of restoring t from g involves differentiations thus making (EE_0) into a nonlinear pde. This pde is not Einstein Equation yet. Recall that in adition to the trivial component there is another O(n)-component in $S^2(\mathfrak{g}_{-1})$, the Ricci tensor Ri. *Einstein equations* (in vacuum and with cosmological term λ) are the *two* conditions: (EE_0) and

$$Ri = 0. \qquad (EE_{ric})$$

A generalization of this example to G-structures associated with certain other CHSSs, flag varieties, and to supermanifolds is considered in [LPS1] and [LP3].

0.4. SF of flag varieties. Contact structures. In heading a) of Proposition 0.2 there are listed all simple Lie algebras of (polynomial or formal) vector fields except those that preserve a contact structure. Recall that a *contact structure* is a maximally nonintegrable distribution of codimension 1, cf. [A].

To consider contact Lie algebra we have to generalize the notion of Cartan prolongation: the tangent space to a point of a manifold with a contact structure possesses a natural structure of the Heisenberg algebra. This is a 2-step nilpotent Lie algebra. Let us consider the general case corresponding to "flag varieties" -- quotients of a simple complex Lie group modulo a parabolic subgroup. (The necessity of such a generalization was very urgent in the classification of simple Lie superalgebra, see [Shch] and [L2], where it first appeared, already superized.)

Given an arbitrary (but $\mathbb{Z}$-graded) nilpotent Lie algebra $\mathfrak{g}_- = \oplus_{0>i\geq -d}\mathfrak{g}_i$ and a Lie subalgebra $\mathfrak{g}_0 \subset \mathfrak{der}\ \mathfrak{g}_-$ which preserves $\mathbb{Z}$-grading of $\mathfrak{g}_-$, define the i-th *prolong* of the pair $(\mathfrak{g}_-, \mathfrak{g}_0)$ for i > 0 to be:

$$\mathfrak{g}_i = (S^*(\mathfrak{g}_-)^*\otimes\mathfrak{g}_0 \cap S^*(\mathfrak{g}_-)^*\otimes\mathfrak{g}_-)_i,$$

where the subscript singles out the component of degree i.Similarly to the above, define $\mathfrak{g}_*$, or rather, $(\mathfrak{g}_-, \mathfrak{g}_0)_*$, as $\oplus_{i\geq -d}\mathfrak{g}_i$; then, by the same reasons as in 0.1, $\mathfrak{g}_*$ is a Lie algebra (subalgebra of $\mathfrak{k}(\dim\mathfrak{g}_-)$ for d = 2 and $\dim\mathfrak{g}_{-2} = 1$) and $H^i(\mathfrak{g}_-; \mathfrak{g}_*)$ is well-defined. $H^i(\mathfrak{g}_-; \mathfrak{g}_*)$ naturally splits into homogeneous components whose degree corresponds to what we will call the *order*. (For the particular case of Lie algebras of depth 2 the obtained bigraded complex was independently and much earlier defined by Tanaka [T] and used in [BS] and [O]. No cohomology was explicitely calculated, however; see calculations in [LPS2] and [LP3].)

The space $H^2(\mathfrak{g}_-; \mathfrak{g}_*)$ is the space of obstructions to flatness. In general case the minimal order of SF is 2-d. For d > 1 we did not establish correspondence between the order of SF and the number of the infinitesimal neighbourhood of a point of a supermanifold with the flat G-structure.

Examples. 1) G* is a simple Lie group, P its parabolic subgroup, G the Levi subgroup of P, $\mathfrak{g}_0$ = Lie(G), $\mathfrak{g}_-$ is the complementary subalgebra to Lie(P) in Lie(G*). The corresponding SF, calculable from the BWB-theorem if $\mathfrak{g}_*$ is finite-dimensional and simple describe for the first time the local geometry of flag varieties other than CHSSs, see [LP3] for details. Here is the simplest example.

2) Let $\mathfrak{g} = \mathfrak{csp}(2n)$, $\mathfrak{g}_{-1} = R(\pi_1; 1)$, $\mathfrak{g}_{-2} = R(0)$; then $\mathfrak{g}_* = \mathfrak{k}(2n+1)$ and

$$C^{k,s}_{\mathfrak{g}_*} = \mathfrak{g}_{k-s}\otimes E^s(\mathfrak{g}_{-1}{}^*) \oplus \mathfrak{g}_{k-s-1}\otimes E^{s-1}(\mathfrak{g}_{-1}{}^*)\otimes\mathfrak{g}_{-2}{}^*.$$

Theorem. *For* $\mathfrak{g}_* = \mathfrak{k}(2n+1)$ *all SF vanish.*

This is a reformulation of the Darboux theorem on a canonical 1-form, actually.

0.5. SF for projective structures. It is also interesting sometimes to calculate $H^2(\mathfrak{g}_-; \mathfrak{h})$ for some $\mathbb{Z}$-graded subalgebras $\mathfrak{h}\subset\mathfrak{g}_*$, such that $\mathfrak{h}_i = \mathfrak{g}_i$ for $i \leq 0$. For example, if $\mathfrak{g} = \mathfrak{gl}(n)$ and $\mathfrak{g}_{-1}$ is its standard (identity) representation we have $\mathfrak{g}_* = \mathfrak{vect}(n)$ and, as we have seen, all SF vanish; but if $\mathfrak{h} = \mathfrak{sl}(n+1) \subset \mathfrak{vect}(n)$ then the corresponding SF are nonzero and provide us with obstructions to integrability of what is called the *projective connection.*

Theorem. 1) *Let* $\mathfrak{g}_* = \mathfrak{vect}(n)$, $\mathfrak{h} = \mathfrak{sl}(n+1)$. *Then SF of order 1 and 2 vanish, SF of order 3 are* R(2,1, 0, ..., 0, -1)

2)*Let* $\mathfrak{g}_* = \mathfrak{k}(2n+1)$, $\mathfrak{h} = \mathfrak{sp}(2n+2)$. *Then SF are* $R(\pi_1+\pi_2; 3)$ *of order 3.*

0.6. Case of simple $\mathfrak{g}_*$ over $\mathbb{R}$.

Example: Nijenhuis tensor. Let $\mathfrak{g}_0 = \mathfrak{gl}(n) \subset \mathfrak{gl}(2n;\mathbb{R})$, $\mathfrak{g}_{-1}$ is the identity module. In this case $\mathfrak{g}_* = \mathfrak{vect}(n)$, however, in seeming contradiction with Theorem 0.1.2, the SF are nonzero. There is no contradiction: now we consider not $\mathbb{C}$-linear maps but $\mathbb{R}$-linear ones.

Theorem. *Nonvanishing SF are of order 1 and constitute the* $\mathfrak{g}_0$*-module* $\overline{\mathfrak{g}_{-1}}\otimes_{\mathbb{C}} E^2{}_{\mathbb{R}}(\mathfrak{g}_{-1}{}^*)$, *where* $g(cv) = \overline{c}V$ *for* $c\in\mathbb{C}$, $g\in\mathfrak{gl}(n)$, $v\in\overline{V}$ *and a* $\mathfrak{gl}(n)$*-module V.*

One of our mottos is: *simple $\mathbb{Z}$-graded Lie superagebras of finite growth* (SZGLSAFGs) *are as good as simple finite-dimensional Lie algebras*; the results obtained for the latter should hold, in some form, for the former. So we calculate

SF on supermanifolds: Plan of campaign

The necessary background on Lie superalgebras and supermanifolds is gathered in a condenced form in [L5], see also [L1, L2]. The above definitions are generalized to Lie superalgebras via Sign Rule.

On the strength of the above examples we must list $\mathbb{Z}$-gradings of SZGLSAFGs of finite depth (recall that a $\mathbb{Z}$-graded Lie (super)algebra of the form $\oplus_{-d\leq i\leq k}\mathfrak{g}_i$ is said to be of *depth* d and *length* k; here d, k >0), calculate projective-like and reduced structures for the above and then go through the list of real forms.

Our theorems are cast in Tables. In Table 1 we set notations. Tables 2 and 3 complement difficult tables of [S]. Table 4 lists all symmetric superspaces of depth 1 of the form G/P with a simple finite-dimensional G. Table 5 lists all hermitian superspaces corresponding to simple loop supergroups different from the obvious examples of loops with values in a hermitian superspace. *Notice that there are 3 series of nonsuper examples.*

We compensate superfluity of exposition by wast bibliography with further results. Let us list some other points of interest in the study of SF on superspaces.

- there is no complete reducibility of the space of SF as $\mathfrak{g}_0$-module;

- Serre's theorem reformulated for superalgebras shows that there are SFs of order >1, see [LPS1];

- faithfulness of $\mathfrak{g}_0$-actions on $\mathfrak{g}_{-1}$ is violated in natural examples of: (a) supergrassmannians of subsuperspaces in an (n,n)-dimensional superspace when the center $\mathfrak{z}$ of $\mathfrak{g}_0$ acts trivially; retain the same definition of Cartan prolongation; the prolong is then the semidirect sum $(\mathfrak{g}_{-1}, \mathfrak{g}_0/\mathfrak{z})_* \ltimes S^*(\mathfrak{g}_{-1}^*)$ with the natural $\mathbb{Z}$-grading and Lie superalgebra structure; notice that the prolong is *not* subalgebra of $\mathfrak{vect}(\dim \mathfrak{g}_{-1})$; (b) the exterior differential d preserving structure.

More precisely, recall that for supermaifolds the good counterpart of differential forms on manifolds are not differential but rather *pseudodifferential and pseudointegrable forms. Pseudodifferential forms* on a supermanifold X are functions on the supermanifold X' associated with the bundle τ^*X obtained from the cotangent one by fiber-wise change of parity. *Differential forms* on X are fiber-wise *polynomial* functions on X'. In particular, if X is a manifold there are no pseudodifferential forms. The *exterior differential* on X is now considered as an odd vector field d on X'. Let $x = (u_1, \dots, u_p, \xi_1, \dots, \xi_q)$ be local coordinates on X, $x_i' = \pi(x_i)$. Then $d = \Sigma x_i'\partial/\partial x_i$ is the familiar coordinate expression of d. The Lie superalgebra $\mathfrak{C}(d) \subset \mathfrak{vect}(m+n/m+n)$, where (m/n) = dim X, -- the Lie superalgebra of vector fields preserving the field d on X' (see definition of the Nijenhuis operator P_4 in [LKW]) -- is neither simple nor transitive and therefore did not draw much attention so far. Still, the corresponding G-structure ($\mathfrak{C}(d) = (\mathfrak{g}_{-1}, \mathfrak{g}_0)_*$, where $\mathfrak{g}_0 = \mathfrak{gl}(k)\ltimes\Pi(\mathfrak{gl}(k))$ and where $\Pi(\mathfrak{gl}(k))$ is abelean and constitutes the kernel of the $\mathfrak{g}_0$-action on $\mathfrak{g}_{-1}$ = id, the standard (identity) representation of $\mathfrak{gl}(k)$) is interesting and natural. Let us call it the *d-preserving structure*. The following theorem justifies pseudocohomology introduced in [LKW].

Theorem. *SFs of the d-preserving structure are 0.*

An interesting counterpart of the d-preserving structure is the odd version of the hamiltonian structure. In order to describe it recall that *pseudointegrable forms* on a supermanifold X are functions on the supermanifold 'X associated with the bundle τX obtained from the *tangent* one by fiber-wise change of parity. Fiber-wise *polynomial* functions on 'X are called polyvector fields on X. (In particular, if X is a manifold there are no pseudointegrable forms.) The *exterior* *differential* on X is now considered as an odd nondegenerate (as a bilinear form) bivector field div on X'. Let $x = (u_1, \ldots, u_p, \xi_1, \ldots, \xi_q)$ be local coordinates on X, $'x_i = \pi(\partial/\partial x_i)$. Then div $= \Sigma\partial^2/\partial x_i'\partial x_i$ is the coordinate expression of the Fourier transform of the exterior differential d with respect to primed variables; the operator is called "div" because it sends a polyvector field on X, i.e. a function on 'X to its divergence. The Lie superalgebra $\mathfrak{aut}$(div) is isomorphic to the Lie superalgebra $\mathfrak{le}$(m+n) which is the simple subalgebra of $\mathfrak{vect}$(n+m|n+m) that preserves a nondegenerate odd differential 2-form $\omega = \Sigma dx_i'dx_i$; an interesting algebra is the superalgebra $\mathfrak{sle}$(m+n) which preserves both div and ω; for both of these Lie superalgebras and their deformations the corresponding SF are calculated in [PS] and [LPS1] .

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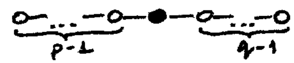
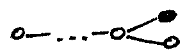
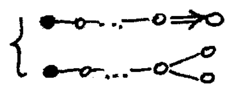
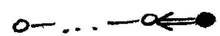
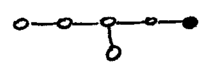
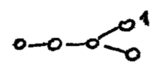
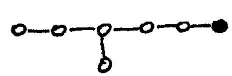
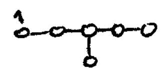
Notations in tables. <u>Everywhere</u> we assume the notational conventions of [S] and definitions adopted there without mentioning this specifically. <u>In Table 1</u> $\mathfrak{s} = (\mathrm{Lie}(S_c)) \otimes \mathbb{C}$, NCHSS is an abbreviation for noncompact hermitian symmetric space, in the diagram of $\mathfrak{s}$ the vertex defining the minimal parabolic subalgebra $\mathfrak{p} = \mathrm{Lie}\ (P)$, such that X can be presented as $(S_c)^{\mathbb{C}}/P$, is shaded. <u>In Table 4</u> we call a homogeneous space G/P, where G is a simple Lie supergroup P its parabolic subsupergroup corresponding to several omitted generators of a Borel subalgebra (description of these generators can be found in [L3, # 31]), of *depth* d and *length* l if such are the depth and length of Lie (G) in the $\mathbb{Z}$-grading compatible with that of Lie(P). Note that all superspaces of Table 4 possess an hermitian structure (hence are of depth 1) except PeGr (no hermitian structure), PeQ (no structure, length 2), $CGr_{0,k}^{0,n}$ and $SCGr_{0,k}^{0,n}$ (no structure, lengths n-k and, resp. n-k-1)

Table 1. Hermitian symmetric spaces

Name of CHSS X	$X = S_c/G_c$	$\mathfrak{s}_0 = (\mathfrak{g}_c)^{\mathbb{C}}$	The diagram of $\mathfrak{s}$	$\mathfrak{s}_{-1} \cong T_0X$	$(S_c)^*$	names of NCHSS
$\mathbb{C}P^n$	$SU(n+1)/U(n)$	$\mathfrak{gl}(n)$		id	$SU(1, n)$	$\mathbb{C}P^n$
Gr_p^{p+q}	$SU(p+q)/S(U(p)\times U(q))$	$\mathfrak{s}(\mathfrak{gl}(p)\oplus\mathfrak{gl}(q))$		$id \otimes id^*$	$SU(p, q)$	$^*Gr_p^{p+q}$
OGr_n	$SO(2n)/U(n)$	$\mathfrak{gl}(n)$		$\Lambda^2 id$	$SO(n, n)$	$^*OGr_n$
Q_n	$SO(n+2)/SO(2)\times SO(n)$	$\mathfrak{co}(n)$		id	$SO(n, 2)$	$^*Q_n$
LGr_n	$Sp(2n)/U(n)$	$\mathfrak{gl}(n)$		$S^2 id$	$Sp(2n;\mathbb{R})$	$^*LGr_n$
$(\mathbb{O}P^2)$	$E_6/SO(10)\times U(1)$	$\mathfrak{co}(10)$			E_6^*	
	$E_7/E_6\times U(1)$	$\mathfrak{ce}6$			E_7^*	

Occasional isomorphisms: $Gr_p^{p+q} \cong Gr_q^{p+q}$, $Q_1 \cong \mathbb{C}P^1$, $Q_3 \cong LGr_2$, $Q_2 \cong S^2\times S^2$, $OGr_2 \cong LGr_1 \cong \mathbb{C}P^1$,
$OGr_3 \cong Gr_3^4$, $Q_4 \cong Gr_2^4$.

Table 2. Dual pairs of homogeneous symmetric superspaces

$(\mathfrak{p})\mathfrak{sl}_r(m\|2n)/\mathfrak{osp}(m,p\|2n)$	$(\mathfrak{p})\mathfrak{su}(m,p\|2n,n)/\mathfrak{osp}(m,p\|2n)$
$(\mathfrak{p})\mathfrak{sl}_r(2m\|2n)/(\mathfrak{p}_r)\mathfrak{s}_r\mathfrak{l}(m\|n)$	$(\mathfrak{p})\mathfrak{su}^*(2m\|2n)/(\mathfrak{p}_r)\mathfrak{s}_r\mathfrak{l}(m\|n)$
$\mathfrak{psl}_r(n\|n)/\mathfrak{pq}_r(n)$	${}^0\mathfrak{pq}(n)/\mathfrak{pq}_r(n)$
$\mathfrak{psl}_r(n\|n)/\mathfrak{spe}_r(n)$	$\mathfrak{supe}(n)/\mathfrak{spe}_r(n)$
$(\mathfrak{p})\mathfrak{su}(m,p\|n,q)/$ $/(\mathfrak{p})\mathfrak{s}(\mathfrak{u}(r+s,r\|t+v,v)\oplus$ $\oplus\,\mathfrak{u}(m-r-s,p-r\|n-t-v,q-t)$	$(\mathfrak{p})\mathfrak{su}(m,p+s-r\|n,v+q)/$ $/(\mathfrak{p})\mathfrak{s}(\mathfrak{u}(r+s,r\|t+v,v)\oplus$ $\oplus\,\mathfrak{u}(m-r-s,p-r\|n-t-v,q-t)$
$(\mathfrak{p})\mathfrak{su}(2n,m\|2n,2q)/\mathfrak{osp}^*(2m\|2n,2q)$	$(\mathfrak{p})\mathfrak{su}^*(2m\|2n)/\mathfrak{osp}^*(2m\|2n,2q)$
$\mathfrak{psu}(m,p\|n,q)/\mathfrak{puq}(n,p)$	$\mathfrak{supe}(n)/\mathfrak{puq}(n,p)$
$\mathfrak{psu}^*(2n\|2n)/\mathfrak{pq}^*(2n)$	${}^0\mathfrak{pq}(n)/\mathfrak{pq}^*(2n)$
$\mathfrak{psu}^*(2n\|2n)/\mathfrak{su}^*(2n)$	$\mathfrak{supe}(2n)/\mathfrak{spe}^*(2n)$
$\mathfrak{osp}(m,p\|2n)/\mathfrak{osp}(s+r,r\|2q)\oplus$ $\oplus\,\mathfrak{osp}(m-r-s,p-s\|2n-2q)$	$\mathfrak{osp}(m,p+s-r\|2n)/\mathfrak{osp}(s+r,r\|2q)\oplus$ $\oplus\,\mathfrak{osp}(m-r-s,p-s\|2n-2q)$
$\mathfrak{osp}(m,p\|2n)/\mathfrak{u}(m/2,p/2\|n,q)$	$\mathfrak{osp}^*(m\|2n,2q)/\mathfrak{u}(m/2,p/2\|n,q)$
$\mathfrak{osp}^*(2m\|2n,2q)/\mathfrak{osp}^*(2p\|2s+2r,2r)\oplus$ $\oplus\,\mathfrak{osp}^*(2m-2p\|2n-2r-2s,2q-2r)$	$\mathfrak{osp}^*(2m\|2n,2q+2s-2r)/$ $/\mathfrak{osp}^*(2p\|2s+2r,2r)\oplus$ $\oplus\,\mathfrak{osp}^*(2m-2p\|2n-2r-2s,2q-2r)$
$\mathfrak{osp}^*(2m\|2n,n)/\mathfrak{osp}_{\mathbb{C}}(m\|n)$	$\mathfrak{osp}(2m\|2n,n)/\mathfrak{osp}_{\mathbb{C}}(m\|n)$
$\mathfrak{psq}_r(2n)/\mathfrak{p}_r\mathfrak{s}_r\mathfrak{q}(n)$	$\mathfrak{psq}^*(2n)/\mathfrak{p}_r\mathfrak{s}_r\mathfrak{q}(n)$
$\mathfrak{psq}_r(2n)/{}^0\mathfrak{p}_r\mathfrak{q}(n)$	$\mathfrak{psq}^*(2n)/{}^0\mathfrak{p}_r\mathfrak{q}(n)$
$\mathfrak{psuq}(m,p)/\mathfrak{ps}(\mathfrak{uq}(r+s,r)\oplus$ $\oplus\,\mathfrak{uq}(m-r-s,p-r))$	$\mathfrak{psuq}(m,p+s-r)/\mathfrak{ps}(\mathfrak{uq}(r+s,r)\oplus$ $\oplus\,\mathfrak{uq}(m-r-s,p-r))$
$\mathfrak{psuq}(m,p)/\mathfrak{pu}(r+s,r\|m-r-s,p-r)$	$\mathfrak{psuq}(m,p+s-r)/\mathfrak{pu}(r+s,r\|m-r-s,p-r)$
$\mathfrak{spe}_r(2n)/\mathfrak{upe}(n)$	$\mathfrak{spe}^*(2n)/\mathfrak{upe}(n)$
$\mathfrak{spe}_r(2n)/\mathfrak{s}_r\mathfrak{pe}(n)$	$\mathfrak{spe}^*(2n)/\mathfrak{s}_r\mathfrak{pe}(n)$
$\mathfrak{sh}(n,p)/\text{Ш}(k,m,p,n)$	$\mathfrak{sh}(n,p+1-k)/\text{Ш}(k,m,p,n)$

Table 3. Selfdual homogeneous symmetric superspaces

$(\mathfrak{p})\mathfrak{su}^*(2m\|2n)/(\mathfrak{p})\mathfrak{s}(\mathfrak{u}^*(2p\|2q)\oplus\mathfrak{u}^*(2m-2p\|2n-2q))$;
$(\mathfrak{p})\mathfrak{sl}_r(m\|2n)/(\mathfrak{p})\mathfrak{s}(\mathfrak{gl}_r(p\|q)\oplus\mathfrak{gl}_r(n-p\|n-q))$;
$(\mathfrak{p})\mathfrak{su}(2m,\ m\|2n,\ n)/\mathfrak{p}_{im}\mathfrak{s}_{im}\mathfrak{l}(m\|n)$; ${}^0\mathfrak{pg}(n)/\mathfrak{p}({}^0\mathfrak{g}(p)\oplus{}^0\mathfrak{g}(n-p))$;
${}^0\mathfrak{pg}(n)/\mathfrak{p}_r\mathfrak{s}_{im}\mathfrak{l}(p\|n-p)$; $\mathfrak{supe}(n)/\mathfrak{s}(\mathfrak{upe}(p)\oplus\mathfrak{upe}(n-p))$; $\mathfrak{supe}(n)/\mathfrak{p}_{im}\mathfrak{s}_r\mathfrak{l}(p\|n-p)$
$\mathfrak{osp}(2m,\ m\|2n)/\mathfrak{gl}_r(m\|n)$; $\mathfrak{osp}^*(2m\|2n,\ n)/\mathfrak{u}^*(m\|n)$; $\mathfrak{psg}_r(n)/\mathfrak{ps}(\mathfrak{g}_r(p)\oplus\mathfrak{g}_r(n-p))$
$\mathfrak{psg}_r(n)/\mathfrak{pgl}_r(p\|-p)$; $\mathfrak{psg}^*(2n)/\mathfrak{ps}(\mathfrak{g}^*(2p)\oplus\mathfrak{g}^*(2n-2p))$; $\mathfrak{psug}(2m,\ m)/\mathfrak{p}_{im}\mathfrak{s}_{im}\mathfrak{g}(m)$;
$\mathfrak{psug}(2m,\ m)/{}^0\mathfrak{p}_{im}\mathfrak{g}(m)$; $\mathfrak{spe}_r(n)/\mathfrak{s}(\mathfrak{pe}_r(n-p)\oplus\mathfrak{pe}_r(p))$; $\mathfrak{spe}_r(n)/\mathfrak{sl}_r(p\|n-p)$
$\mathfrak{spe}^*(2n)/\mathfrak{s}(\mathfrak{pe}^*(2p)\oplus\mathfrak{pe}^*(2n-2p))$; $\mathfrak{spe}^*(2n)/\mathfrak{su}^*(2p\|2n-2p)$; $\mathfrak{sh}(2n,\ n)/\text{Ц}_r(n)$

Table 4. Classical superspaces of depth 1

$\mathfrak{g}$	$\mathfrak{g}_0$	$\mathfrak{g}_{-1}$	Interpretation	Underlying domain	Name of the superdomain
$\mathfrak{sl}(m\vert n)$	$\mathfrak{s}(\mathfrak{gl}(p\vert q)\oplus\mathfrak{gl}(m-p\vert n-q))$	$id\otimes id^*$	Supergrassmannian of the $(p\vert q)$-dimensional subsuperspaces in $\mathbb{C}^{m\vert n}$-dimensional one	$Gr_p^m\times Gr_q^n$	$Gr_{p,q}^{m,n}$
$\mathfrak{psl}(m\vert n)$	$\mathfrak{ps}(\mathfrak{gl}(p\vert p)\oplus\mathfrak{gl}(m-p\vert n-q))$	$id\otimes id^*$	Same for $m=n$, $p=q$	$Gr_p^m\times Gr_p^m$	$Gr_{p,p}^{m,m}$
$\mathfrak{osp}(m\vert 2n)$	$\mathfrak{cosp}(m-2\vert 2n)$	id	Superquadric of $(1\vert 0)$-dimensional isotropic with respect to the nondege-nerate even form lines in $\mathbb{C}^{m\vert n}$	Q_{m-2}	$Q_{m-2,n}$
$\mathfrak{osp}(2m\vert 2n)$	$\mathfrak{gl}(m\vert n)$	$E^2\ id$	Ortholagrangean supergrassmannian of $(m\vert m)$-dimensional isotropic with respect to the nondegenerate even form subsuperspaces in $\mathbb{C}^{2m\vert n}$	$OGr_m\times {}^*LGr_n$ ${}^*OGr_m\times LGr_n$	$OLGr_{m,n}$
$\mathfrak{sq}(n)$ $\mathfrak{psq}(n)$	$\mathfrak{s}(\mathfrak{q}(p)\oplus\mathfrak{q}(n-p))$ $\mathfrak{ps}(\mathfrak{q}(p)\oplus\mathfrak{q}(n-p))$	$irr(id\otimes id^*)$	Queergrassmannian of q-symmetric $(p\vert p)$-dimensional subsuperspace in $\mathbb{C}^{n\vert n}$	Gr_p^n	QGr_p^n
$\mathfrak{pe}(n)$ $\mathfrak{spe}(n)$	$\mathfrak{cpe}(n-1)$ $\mathfrak{cspe}(n-1)$	id	Odd superquadric of $(1\vert 0)$-dimensional isotropic with respect to the nondegenerate odd form lines in $\mathbb{C}^{n\vert n}$	$\mathbb{CP}^{n-1}$	PeQ_{n-1}
$\mathfrak{pe}(n)$ $(\mathfrak{spe}(n))$	$\mathfrak{gl}(p\vert n-p)$ $(\mathfrak{sl}(p\vert n-p))$	$\pi(S^2\ (id))$ or $\pi(E^2\ (id))$	Odd lagrangean supergrassmannian of $(p\vert n-p)$-dimensional (and with a fixed volume for $\mathfrak{spe}$) subsuperspaces in $\mathbb{C}^{n\vert n}$ isotropic with respect to the odd symmetric or skewsymmetric form	Gr_p^n	$PeGr_p^n$
$\mathfrak{vect}(0\vert n)$	$\mathfrak{vect}(0\vert n-k)\oplus\mathfrak{gl}(k;\Lambda(n-k))$	$\Lambda(k)\otimes\pi(id)$	Curved supergrassmanian of $(0\vert 1)$-dimensional subsupermanifolds in $\mathbb{C}^{0\vert n}$		$CGr_{0,k}^{0,n}$
$\mathfrak{svect}(0\vert n)$	$\mathfrak{vect}(0\vert n-k)\oplus\mathfrak{sl}(k;\Lambda(n-k))$	$\pi(Vol)$ if $k=1$	Same with volume elements preserved in the sub- and ambient supermanifolds		$SCGr_{0,k}^{0,n}$
$\mathfrak{h}(0\vert m)$ $\mathfrak{sh}(m)$	$\mathfrak{h}(0\vert m-2)\oplus\Lambda(m-2)\bullet z$ $\mathfrak{sh}(m-2)\oplus\Lambda(m-2)\bullet z$	$\pi(id)$	Curved superquadeic of $(0\vert 1)$-dimensional isotropic with respect to the (partly) split symmetric form subsupermanifolds in $\mathbb{C}^{0\vert m}$		$CQ_{m-2,0}$

$\mathfrak{d}(\alpha)=$ $=\mathfrak{osp}(4\|2)_\alpha$	$\mathfrak{cosp}(2\|2)\cong(\mathfrak{gl}(2\|1))$	id	$\mathbb{CP}^1\times\mathbb{CP}^1$
$\mathfrak{ab}(3)$	$\mathfrak{cosp}(2\|4)$ $\mathfrak{cd}(2)$	$L_{3\varepsilon_1}$	$\mathbb{CP}^1\times Q_5$

Table 5. Gradings of twisted loop (super)algebras corresponding to hermitian superdomains

$\mathfrak{g}_\varphi^{(m)}$	φ	*grading elements from* $\mathfrak{h}$	$(\mathfrak{g}_\varphi^{(m)})_0$
$\mathfrak{sl}(2m/2n)^{(2)}$	$(-st)\circ Ad\ diag(\pi_{2m},J_{2n})$	$diag(1_m,-1_m,1_n,-1_n)$	$\mathfrak{sl}(m/n)^{(1)}$
$\mathfrak{sl}(2m)^{(2)}$	$(-t)\circ Ad\ (\pi_{2m})$		$\mathfrak{sl}(m)^{(1)}$
$\mathfrak{sl}(2n)^{(2)}$	$(t)\circ Ad\ (J_{2n})$		$\mathfrak{sl}(n)^{(1)}$
$\mathfrak{sl}(n/n)^{(2)}$	π	$diag(1_p,0_{n-p},1_p,0_{n-p})$	$\mathfrak{s}(\mathfrak{gl}(p/p)^{(2)}{}_\pi\oplus\mathfrak{gl}(n-p/n-p)^{(2)}{}_\pi)$
$\mathfrak{sl}(n/n)^{(2)}$	$\pi\circ(-st)$	$diag(1_p,-1_{n-p},-1_p,1_{n-p})$	$\mathfrak{s}(\mathfrak{gl}(p/p)^{(2)}{}_{\pi\circ(-st)}\oplus\mathfrak{gl}(n-p/n-p)^{(2)}{}_{\pi\circ(-st)})$
$\mathfrak{osp}(2m/2n)^{(2)}$	$\varphi_{m,n}Ad\ diag(1_{2m-1},1,1_{2n})$	$diag(2J_2,\ O_{2(m+n-1)})$	$(\mathfrak{cosp}(2m-2/2n))^{(1)}{}_{\varphi_{m-1,n}}$
$\mathfrak{o}(2m)^{(2)}$			$(\mathfrak{co}(2m-2))^{(1)}$
$\mathfrak{psq}(2n)^{(4)}$	$(-st)\circ\sigma_i$	$diag(J_{2n},J_{2n})$	$\mathfrak{psq}(n)^{(2)}{}_{\delta_{-1}}$
$\mathfrak{sh}(2n)^{(2)}$	A	$H_{\xi_2\xi_3}$	$(\mathfrak{sh}(2n-2)\oplus\Lambda(2n-2))^{(2)}{}_A$
$\mathfrak{psq}(n)^{(2)}$	σ_{-1}	$diag(1_p,0_{n-p},1_p,0_{n-p})$	$\mathfrak{ps}(\mathfrak{q}(p)^{(2)}{}_{\delta_{-1}}\oplus\mathfrak{q}(n-p)^{(2)}{}_{\delta_{-1}})$