(*superpotential*) [2]. An exclusion can be only lowest level of one of $H_{\pm}$ which energy vanishes exactly. These properties allow us to find the exact spectrum of those one-dimentional ordinary potentials which has the property of shape-invariance [1], i.e. the potentials $V_{\pm}(x)$ differ only by parameter values. It turned out that all *exactly solvable* potentials of ordinary quantum mechanics are shape-invariant and their spectrum can be found by means of elementary calculations [1].

**Example**. Let $W(x,a) = a\tanh x$, then $H_{\pm}(a) = \frac{1}{2}\left(p^2 - a(a \mp 1)\cosh^{-2}(x)\right) + \frac{a^2}{2}$ and $H_{+}(a) = H_{-}(a-1) + a - \frac{1}{2}$. If $a > 1$, then the lowest lewel of $H_{-}(a-1)$ vanishes (as for any nondegenerated *superpotential* $W(x)$), hence the lowest lewel of $H_{+}(a)$ is equal to $a - \frac{1}{2}$. This procedure can be repeated $n$ times for $a > n$, giving the whole spectrum of ordinary Hamiltonian $H_{-}(a)$ as $E_n^{(-)} = -\frac{(a-n)^2}{2} + \frac{a^2}{2}$ which differs from the well-known spectrum $E_n = -\frac{(a-n)^2}{2}$ of $V(x,a) = -\frac{1}{2}a(a+1)\cosh^{-2}(x)$ by only additive constant $\frac{a^2}{2}$.

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**EXCEPTIONAL SUPERALGEBRA** — A simple **Lie superalgebra** that is not a member of a series, like, e.g., $G_2$ and unlike $\mathfrak{sl}(n)$.

A simple finite dimensional **Lie algebra** over complex numbers constitute *classical series* $\mathfrak{sl}$, $\mathfrak{o}$ and $\mathfrak{sp}$ there are also five exceptional **Lie superalgebras** S. Lie discovered in his study of symmetries of differential equations; they have no simple description ([10]) but are related, e.g., to simple singularities in mathematics and *string theory* in physics [1,9].

Passage to **Kac–Moody algebras** (or *loop algebras* with values in the above finite dimensional ones) does not bring new examples [2].

Among simple **Lie algebras** of vector fields there are only series and no exceptions; the class of "*stringy*" *simple Lie algebras*, on the contrary, consists only of the *centerless Virasoro algebra.*

Passing to *simple Lie superalgebras* we find the following exceptional and often unexpected algebras: (1) 2 individual ones and a family with a continuous parameter among finite dimensional algebras ([3,4]); (2) several new *Kac–Moody superalgebras* ([5]); (3) 15 superalgebras of vector fields with polynomial coefficients [6] pertaining, perhaps, to the *Standard Model* [7]; (4) 5 "*stringy*" (superconformal) superalgebras ([8]).

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**EXCHANGE STATISTICS** — A statistics of two identical particles 1 and 2, that is determined by a double exchange process $1 \to 2$ and $2 \to 1$. Every such exchanging yields an arbitrary phase factor $q$, where $q$ is a complex number on the unit circle, i.e. $|q| = 1$. The corresponding particles are said to be in general **anyons** [1]. Obviously $q = 1$ for bosons and $q = -1$ for fermions. The name "quons" is also used for these particles [2]. The name "**anyons**" is then reserved to the case when $q$ is the $m$-th root of unity [3]. The exchange **braid statistics** [4] for a system of $N$ identical hard core particles is defined by the relation

$$x^i\ x^j = \Sigma_{k,l}^N\ B_{kl}^{ij}\ x^k\ x^l,$$

where $B : E \otimes E \to E \otimes E$ satisfy the quantum *Yang–Baxter equation*

$$(B \otimes id)(id \otimes B)(B \otimes id) = (id \otimes B)(B \otimes id)(id \otimes B),$$

and $B^2 = id_E$.

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